Semester 2 (Unit 1 & Unit 2) Examination, 2017

Question/Answer Booklet

MATHEMATICS SPECIALIST

Section Two: Calculator-assumed

Student Name/Number: _____

Teacher Name: _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time for this section: 100 minutes

Materials required/recommended for this section

To be provided by the supervisor: This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate:

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	9	50	50	35
Section Two: Calculator-assumed	13	13	100	100	65
					100

Instructions to candidates

1. The rules for the conduct of School exams are detailed in the _____*School/College assessment policy*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

Section Two: Calculator-assumed **65% (100 Marks)**

This section has **15** questions. Answer **all** questions. Write your answers in the spaces provided. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Suggested working time: **100 minutes**.

Question 10 **(7 marks)**

(a) If A, B and C are all 2×2 matrices that all have inverses and $C = A - BC$, determine the matrix C given that A = $\begin{bmatrix} 1 & -1 \\ 5 & -6 \end{bmatrix}$ and B = $\begin{bmatrix} 3 & 1 \\ 5 & 5 \end{bmatrix}$. (4 marks)

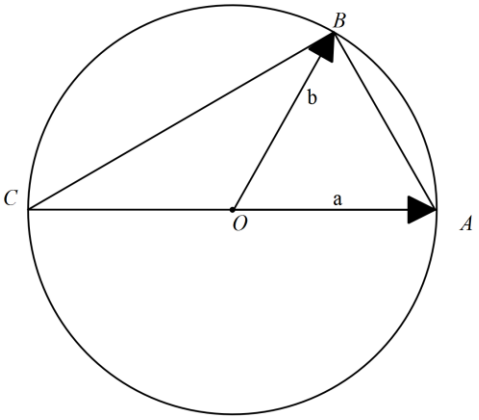
(b) Determine the value of x for the following singular matrix. (3 marks)

$$\begin{bmatrix} 3 & x+1 \\ 2 & -x \end{bmatrix}$$

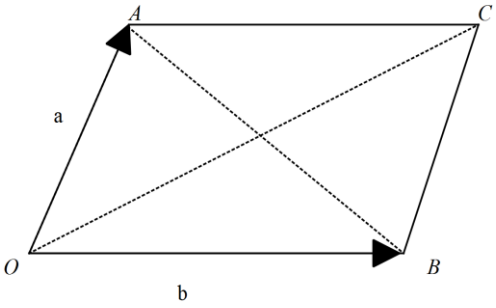
Question 11 **(10 marks)**

Using vector methods, prove the following:

(a) The angle in a semi-circle is a right angle. (5 marks)



(b) The squares of the lengths of the diagonals of a parallelogram equals the sum of the squares of each side length. (5 marks)



Question 12 **(11 marks)**

(a) Given that $\left|\underset{\sim}{a}\right| = 7$, $\left|\underset{\sim}{b}\right| = 9$ and $\underset{\sim}{a}.\underset{\sim}{b} = 31.5$, determine the following:

(i) $\underset{\sim}{a}.\underset{\sim}{a}$. (1 mark)

(ii) The angle between $\underset{\sim}{a}$ and $\underset{\sim}{b}$. (2 marks)

(iii) The scalar projection of $\underset{\sim}{a}$ onto $\underset{\sim}{b}$. (2 marks)

(b) Let $\underset{\sim}{d} = 21i - 7j$, $\underset{\sim}{w} = 2i - \mu j$ and $\underset{\sim}{v} = \lambda i + 15j$.

(i) Determine μ given that $\underset{\sim}{d}$ and $\underset{\sim}{w}$ are perpendicular. (3 marks)

(ii) Determine λ given that $\underset{\sim}{v}$ and d are parallel. (3 marks)

Question 13 **(10marks)**

(a) The angle between $\underset{\sim}{c}$ and $\underset{\sim}{d}$ is $\frac{\pi}{6}$ radians. If $\underset{\sim}{c} = 2i + j$ and $\underset{\sim}{d} = 5i - xj$ determine the two possible values of x to 2 decimal places. (4 marks)

(b) A rhombus OACB is formed with vertex O at the origin (0,0), A at (1,7) and B at (5,5). Calculate the position vector for vertex C the longer diagonal of the rhombus. (2marks)

(c) Use the diagonals to prove your resulting shape OACB is in fact a rhombus. (4 marks)

Question 14 **(9 marks)**

(a) The number 37 can have the digits reversed as 73. The difference between 73 and 37 is 36.

(i) Repeat this for other **two digit numbers** (ie) reverse their digits and then find their difference. (2 marks)

(ii) From your results formulate a conjecture. (1 marks)

(iii) Is your conjecture true or false? If false give an example, if true give an algebraic proof. (3 marks)

(b) Prove by contradiction that there is no smallest rational number greater than zero. (3 marks)

Question 15 **(6 marks)**

If $S = \{1, 2, 3, ..., 99, 100\}$, determine the number of integers in S which are NOT odd NOR divisible by 3 NOR 5 using the inclusion-exclusion principle.

Hint:
$Let\ \mathrm{A} = \{\text{Odd integers}\}\ B = \{\text{Integers divisible by } 3\}$ and $C = \{\text{Integers divisible by } 5\}$
where A, B and $C \in S$

Question 16 **(8marks)**

(a) If $4\sin\theta\cos^2\frac{\theta}{2} = A\sin B\theta + B\sin A\theta$, determine the values of A and B. (4 marks)

(b) Solve the equation $\sin 5x - \sin x = \sin 2x$ for $0 \le x \le \pi$, algebraically. (4 marks)

Question 17 **(9 marks)**

(c) If the graph of $y = \sin x$ is dilated by a factor of $\frac{1}{2}$ parallel to both axes and translated 2 units downward, what is the resulting equation? (2 marks)

(d) Consider the graph of $y = \sec(3x - \pi)$ over the domain $0 \le x \le \pi$, state the vertical asymptotes, expressed in terms of π. (2 marks)

(e) Express $\tan x$ in terms of $\tan\frac{1}{2}x$ and hence determine an exact value of $\tan\frac{5\pi}{8}$, showing all working steps. [Hint: Use $\tan\frac{5\pi}{4} = 1$ and the quadratic formula.] (5 marks)

Question 18 **(5 marks)**

(a) In a circle, chords of equal length subtend equal angles at the centre.

State the converse of the above statement. (1 mark)

(b) If an angle is subtended by a semi-circle then the angle is a right angle.

(i) Write the contrapositive statement. (1 mark)

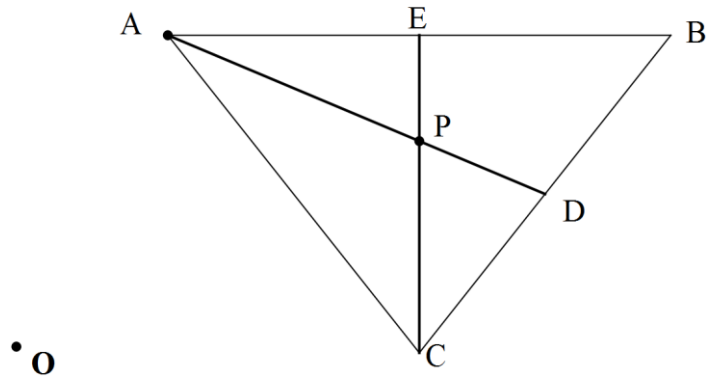
(ii) Is the contrapositive statement true? Justify. (2 marks)

(c) Provide a counter example to disprove the statement, $a > b \Rightarrow \frac{1}{a} < \frac{1}{b} \quad \forall\ a,b \in \mathbf{R}$

(2 marks)

Question 19 **(5 marks)**

In the triangle ABC drawn below, D is the midpoint of $\overline{CB}$, E is the midpoint of $\overline{AB}$ and O may be taken as the Origin to reference position vectors. P is the point of intersection of $\overline{CE}$ and $\overline{AD}$.



Let $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ denote the position vectors of A, B and C.

(a) Determine $\overrightarrow{CE}$ in terms of $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$. (2 marks)

(b) Assuming that P is the point of trisection of the Median $\overline{CE}$, show that $\overrightarrow{AP} = \frac{1}{3}(\mathbf{b} + \mathbf{c} - 2\mathbf{a})$. (3 marks)

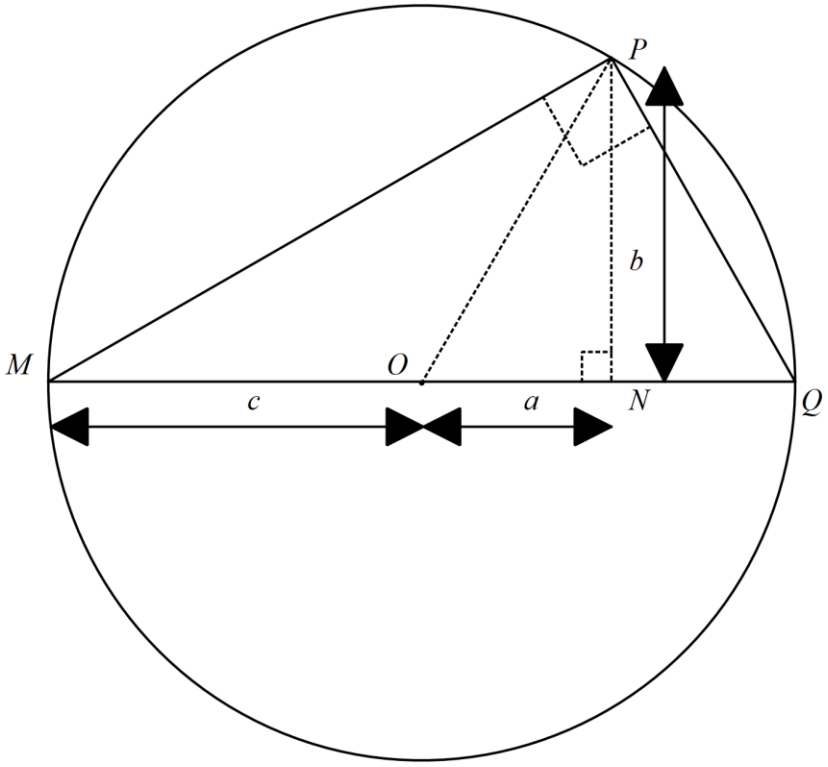
Question 20 **(5 marks)**

By considering an anticlockwise rotation of angle A followed by a clockwise rotation B about the origin, prove using the transformation matrix $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ that

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

Question 21 **(5 marks)**

The angle $\angle MPQ$ is a right angle inside a circle of center O.

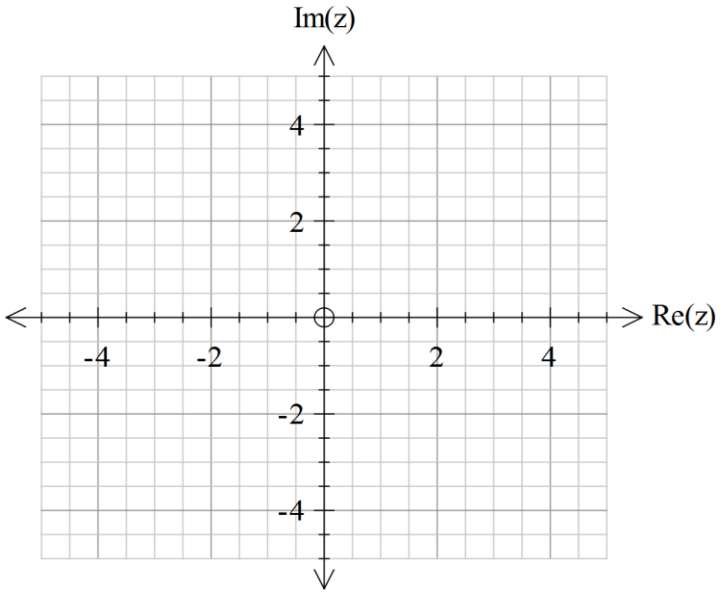


(a) Prove that triangles ΔMPN and ΔPQN are similar. (2 marks)
(Hint: Look at pairs of angles that add up to 90°)

(b) By considering corresponding side length ratios of the above triangles, prove the Pythagorean theorem $c^2 = a^2 + b^2$. (3 marks)

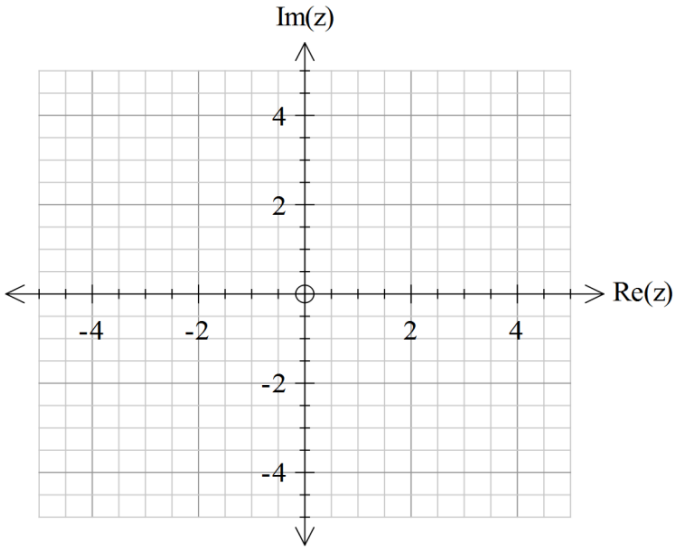
Question 22 **(10 marks)**

a) With $z = 1 + i$, display $z, iz, i^2 z\ i^3 z$ as vectors on the single Argand diagram below. (4 marks)



b) Explain the effect on the argument and modulus of z each time we multiply by i. (3 marks)

c) Given $w = -1 + 5i$ plot w, $i^6 w$ and $i^{13} w$ on the axes below. (3 marks)



END OF PAPER

Additional working space

Question number: _____

Acknowledgements

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